1887MNRAS..47..480M

Epoch.	Observer.	θ_o	θ_c	$\theta_o-\theta_c$	ρ_o	ρ_c	$\rho_o-\rho_c$
1872·33	Wilson & Seabroke	64·8	64·25	+0·55	2·30	2·37	−0·07
872·37	Talmage	67·3	64·27	+3·03	—	2·37	—
1873·23	Wilson & Seabroke	64·0	64·80	−0·80	2·05	2·38	−0·33
1873·35	„	65·0	64·87	+0·13	2·00	2·38	−0·38
1874·32	„	65·5	65·45	+0·05	2·15	2·40	−0·25
1874·32	Talmage	69·8	65·45	+4·35	1·08?	2·40	−1·32?
1874·41	Wilson & Seabroke	64·2	65·50	−1·30	2·16	2·40	−0·24
1875·31	Schiaparelli	66·6	66·03	+0·57	2·00	2·42	−0·42
1876·36	Wilson & Seabroke	67·2	66·64	+0·56	2·15	2·44	−0·29
1876·41	„	66·5	66·67	−0·17	2·21	2·45	−0·24
1879·40	Hall	68·9	68·40	+0·50	2·33	2·50	−0·17
1879·470	Schiaparelli	67·98	68·40	−0·42	2·462	2·50	−0·038
1882·467	„	69·38	70·00	−0·62	2·314	2·55	−0·236
1883·21	Engelmann	70·44	70·43	+0·01	2·623	2·56	+0·063
1887·213	Tarrant	72·44	72·52	−0·08	2·62	2·62	0·00
1887·356	Young	72·75	72·59	+0·16	2·79	2·62	+0·17

I have computed the following short ephemeris:—

Date.	Position Angle.	Distance.
1890·0	73·91°	2·73″
1895·0	76·32	2·78
1900·0	78·64	2·83

On the Formulæ for Correcting Approximate Elements of the Orbits of Binary Stars. By A. Marth.

The present communication, a supplement of the paper in the April number of the *Monthly Notices*, pp. 333–46, is intended to point out the alterations and modifications of the formulæ there given, which render them applicable to the orbits of binary stars, and to offer some remarks connected therewith.

While in the case of the orbit of a satellite belonging to the solar system the apparent orbit, or the projection of the true orbit on a plane perpendicular to the line of vision, changes continually with the changing position of the Earth relative to the planet, in the case of a binary system the plane of projection remains unaltered, and may therefore properly be adopted as the plane to which the plane of the orbit is referred.

Let $☊_0$ denote the position-angle of the line of nodes of the two planes, reckoned from the declination-circle passing through the star at the time t_0, ☊ the same angle reckoned from the declination-circle at the time t, so that

$$☊ = ☊_0 + 0^\circ{\cdot}00557 \sin\alpha \,.\, \sec\delta \,.\, (t-t_0).$$

1887MNRAS..47..480M

Let i be the inclination of the two planes, and let it be taken between 0° and 90°, if the apparent motion of the comes is in the direction of *in*creasing position-angles, and between 90° and 180°, if the motion is in the direction of *de*creasing position-angles, so that there may be no need to make a distinction between direct and retrograde motion in the orbit.

Further, let ω denote the angle between the node and the lower apsis in the direction of the real motion in the orbit; $e = \sin\phi$ the eccentricity; a the major semi-axis of the orbit, expressed in seconds of arc; $a\rho$ the radius vector, μ, ϵ, v the mean, eccentric, and true anomaly of the comes at the time t; μ_0 its mean anomaly at the time t_0; n its yearly mean orbital motion.

The true anomaly v of the comes and its radius vector ρ, expressed in parts of a, being found by the usual formulæ

$$\epsilon - e^{\circ} \,.\, \sin\epsilon = \mu = \mu_0 + n(t - t_0) \qquad e^{\circ} = 57^{\circ}{\cdot}296 \,.\, e,$$

$$\rho \sin v = \sin\epsilon \,.\, \cos\phi$$

$$\rho \cos v = \cos\epsilon - e,$$

the values of the position-angle p and of the distance s of the comes for the time t are found by

$$\sin\sigma \sin(p - \text{☊}) = \sin(v + \omega)\cos i$$

$$\sin\sigma \cos(p - \text{☊}) = \cos(v + \omega)$$

$$s = a\rho \,.\, \sin\sigma.$$

Formulæ for correcting the assumed elements by means of equations of condition between observed and computed position-angles have been published by Mädler (*Astron. Nachr.*, vol. xvi. p. 36); also by Schur (*Astron. Nachr.*, vol. lxxi. p. 24), and perhaps in other papers, not at present within my reach. To be really efficient, such formulæ require that the assumed elements should be fairly approximate, but a rather wide field is left for their useful application. In the case of elements of questionable character, or even of first attempts, the formation of the equations of condition ought not to be neglected, for their mere aspect may prove to the computer the inadequacy of the observations at his disposal for a fair determination of the orbit; or if it does not prevent the publication of premature results of very doubtful value, it will at least prevent their undue acceptance by readers competent to form their own judgment.

Introducing for greater simplicity, as in the case of the orbit of a satellite belonging to the solar system—

$$\sin\sigma \sin\tau = +\cos i$$

$$\sin\sigma \cos\tau = -\sin i \cos(v + \omega)$$

$$\cos\sigma = +\sin i \sin(v + \omega),$$

1887MNRAS..47..480M

the differentiation of the equations for p and s gives

$$\sin\sigma dp = \sin\tau \,.\, d(v+\omega) + \cos\tau \,.\, \sin(v+\omega) \,.\, di + \sin\sigma d\text{☊}$$

$$d\sigma = \cos\tau \,.\, d(v+\omega) - \sin\tau \,.\, \sin(v+\omega) \,.\, di.$$

If e and i, eccentricity and inclination, are not of small amount, the equations of condition between the differences of the observed and computed position-angles and distances and the variations of the assumed elements will be

$$\begin{aligned}
\rho\sin\sigma \,.\, \delta p = {} & +\sin\tau\frac{\cos\phi}{\rho} \,.\, [\delta\mu_0 + (t-t_0)\delta n] \\
& +\rho\sin\tau \,.\, \delta\omega \\
& +\sin\tau(\sin v\cos\phi + \sin\epsilon) \,.\, \delta\phi \\
& +\rho\cos\tau \,.\, \sin(v+\omega) \,.\, \delta i \\
& +\rho\sin\sigma \,.\, \delta\text{☊}
\end{aligned}$$

$$\begin{aligned}
\frac{57{\cdot}3}{a} \,.\, \delta s = {} & +(\cos\sigma\cos\tau \,.\, \frac{\cos\phi}{\rho} + \tan\phi\sin\sigma\sin v) \,.\, (\delta\mu_0 + (t-t_0)\delta n) \\
& +\rho\cos\sigma\cos\tau \,.\, \delta\omega \\
& +[\cos\sigma\cos\tau(\sin v\cos\phi + \sin\epsilon) - \sin\sigma\cos v\cos\phi) \,.\, \delta\phi \\
& -\rho\cos\sigma\sin\tau \,.\, \sin(v+\omega) \,.\, \delta i \\
& +\rho\sin\sigma \,.\, \frac{57{\cdot}3}{a}\delta a.
\end{aligned}$$

In cases of moderate or small eccentricity it will be safer to substitute

$$\begin{aligned}
\rho\sin\sigma \,.\, \delta p = {} & +\sin\tau \,.\, (1+e\cos v) \,.\, [\sec\phi\ \delta(\mu_0+\omega) + (t-t_0) \,.\, \sec\phi \,.\, \delta n] \\
& -\sin\tau \,.\, (\cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi) \,.\, 57{\cdot}3\,(\tan\phi + \delta e)\sin\delta\omega \\
& +\sin\tau \,.\, (\sin v + \sin\epsilon\sec\phi) \,.\, 57{\cdot}3\,[(\tan\phi + \delta e)\cos\delta\omega - \tan\phi] \\
& +\rho\cos\tau \,.\, \sin(v+\omega) \,.\, \delta i \\
& +\rho\sin\sigma \,.\, \delta\text{☊}
\end{aligned}$$

$$\begin{aligned}
\frac{57{\cdot}3}{a} \,.\, \delta s = {} & +[\cos\sigma\cos\tau \,.\, (1+e\cos v) + e\sin\sigma\sin v][\sec\phi\ \delta(\mu_0+\omega) \\
& \qquad\qquad + (t-t_0)\sec\phi \,.\, \delta n] \\
& -[\cos\sigma\cos\tau \,.\, \cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi) + \sin\sigma\sin v] \\
& \qquad\qquad \,.\, 57{\cdot}3\,(\tan\phi + \delta e)\sin\delta\omega \\
& +[\cos\sigma\cos\tau \,.\, (\sin v + \sin\epsilon\sec\phi) - \sin\sigma\cos v] \\
& \qquad\qquad \,.\, 57{\cdot}3\,[(\tan\phi + \delta e)\cos\delta\omega - \tan\phi] \\
& -\rho\cos\sigma\sin\tau \,.\, \sin(v+\omega) \,.\, \delta i \\
& +\rho\sin\sigma \,.\, \frac{57{\cdot}3}{a}\delta a.
\end{aligned}$$

1887MNRAS..47..480M

If the assumed orbit is circular and the orbital longitude of the comes from the node is $= u$, the equations become

$$\begin{aligned}\sin\sigma\delta p = &+\sin\tau\,.\,[\delta u_0+(t-t_0)\,\delta n]\\&-\sin\tau\,.\,\cos u\,.\,2\phi\sin\omega\\&+\sin\tau\,.\,\sin u\,.\,2\phi\cos\omega\\&+\cos\tau\,.\,\sin u\,.\,\delta i\\&+\sin\sigma\,.\,\delta\,\text{☊}\end{aligned}$$

$$\begin{aligned}\frac{57{\cdot}3}{a}\,\delta s = &+\cos\sigma\cos\tau\,[\delta u_0+(t-t_0)\,\delta n]\\&-(\cos\sigma\cos\tau\,.\,\cos u+\tfrac{1}{2}\sin\sigma\,.\,\sin u)\,.\,2\phi\sin\omega\\&+(\cos\sigma\cos\tau\,.\,\sin u+\tfrac{1}{2}\sin\sigma\,.\,\cos u)\,.\,2\phi\cos\omega\\&-\cos\sigma\,.\,\sin\tau\,.\,\sin u\,.\,\delta i\\&+\sin\sigma\,.\,\frac{57{\cdot}3}{a}\,\delta a.\end{aligned}$$

In case the inclination of the orbit to the plane of projection is not considerable, the determination of the line of nodes becomes uncertain, and this uncertainty develops with the decrease of inclination rapidly into indeterminateness. The uncertainty of fixing the position of the plane of the orbit may be easily estimated by the reflection that in order that the maximum effect of the inclination upon the position-angles may reach the amount of

0°·5 1°·0 1°·5 2°·0

the inclination must amount to

10°·7 15°·1 18°·4 21°·2

the corresponding maximum of the foreshortening of the radius vector being

0·017 ·034 ·051 0·067

or 1 : 58 29 20 15.

A fair determination of ☊ and i and consequently of ω being, under such circumstances, not practicable, they cannot serve directly as elements of the orbit, but ought to be supplanted by such functions or such other elements as will practically represent whatever can be fairly established concerning the position of the orbit, without erring either on the one side by a display of untrustworthy figures, or, on the other, by the assumption or suggestion of entire uncertainty.

The proper selection of the most suitable elements for the purpose does not appear to me doubtful. The choice will be different, according as the eccentricity of the orbit is considerable or not.

In case the orbit is so eccentric that the place of the chief star in the projected orbit is considerably distant from the centre,

1887MNRAS..47..480M

the substitution of the position-angle P of the periastre for that of the line of nodes as an element recommends itself at once, and the selection of

$$\tan\nu = \frac{\sin^2 i \sin\omega \cos\omega}{1 - \sin^2 i \sin^2\omega} \text{ and of } k = \frac{\cos i}{1 - \sin^2 i \sin^2\omega}$$

as the two other new elements will be found a suitable one in the course of some simple considerations.

If in the equations for finding p and $\sin\sigma$

$$\sin\sigma \sin(p - \text{☊}) = \sin(v + \omega) \cos i$$
$$\sin\sigma \cos(p - \text{☊}) = \cos(v + \omega)$$

the true anomaly v is put $= 0°$, they supply the position-angle P of the periastre and the corresponding value of $\sin\sigma$, say $\sin\sigma_0$:

$$\sin\sigma_0 \sin(\mathrm{P} - \text{☊}) = \sin\omega \cos i$$
$$\sin\sigma_0 \cos(\mathrm{P} - \text{☊}) = \cos\omega$$

and thereby

$$\sin^2\sigma_0 = 1 - \sin^2 i \sin^2\omega.$$

The substitution of these values in the preceding equations gives

$$\sin\sigma_0 . \sin\sigma \sin(p - \mathrm{P}) = \sin v . \cos i$$
$$\sin\sigma_0 . \sin\sigma \cos(p - \mathrm{P}) = \cos v . (1 - \sin^2 i \sin^2\omega) - \sin v . \sin^2 i \sin\omega \cos\omega$$
$$\text{or } \frac{\sin\sigma}{\sin\sigma_0} . \sin(p - \mathrm{P}) = \sin v . \frac{\cos i}{1 - \sin^2 i \sin^2\omega}$$
$$\frac{\sin\sigma}{\sin\sigma_0} . \cos(p - \mathrm{P}) = \cos v - \sin v . \frac{\sin^2 i \sin\omega \cos\omega}{1 - \sin^2 i \sin^2\omega}.$$

By the introduction of the position-angle

$$\mathrm{P} = \mathrm{P}_0 + 0°{\cdot}00557 \sin\alpha . \sec\delta . (t - t_0) \text{ of the periastre,}$$

and of

$$\frac{\cos i}{1 - \sin^2 i \sin^2\omega} = k \text{ and } \frac{\sin^2 i . \sin\omega \cos\omega}{1 - \sin^2 i \sin^2\omega} = \tan\nu$$

as the new elements, the equations for finding p and s become, if (s) is written instead of

$$\frac{\rho \sin\sigma}{\sin\sigma_0}$$

and (a) instead of $a \sin\sigma_0$ or $a\sqrt{1 - \sin^2 i \sin^2\omega}$,

$$(s) \sin(p - \mathrm{P}) = \rho \sin v . k \qquad = \sin\epsilon . k \cos\phi$$
$$(s) \cos(p - \mathrm{P}) = \rho \cos v - \rho \sin v . \tan\nu = \cos\epsilon - \sin\epsilon . \tan\nu . \cos\phi - e$$
$$s = (a) . (s)$$

If the plane of the orbit coincides with the plane of projection, the value of $\tan\nu$ is $= 0$ and that of $k = +1$ or -1 according as the apparent motion of the comes is in the direction of *in-*

1887MNRAS..47..480M

creasing or *de*creasing position-angles. The values of tan ν and of the deviation of k from ± 1, which a careful discussion of the observations furnishes, represent fully whatever the observations really indicate about the inclination of the two planes and its direction.

The differentiation of the last equations gives

$$\begin{aligned}(s)\,dp &= \frac{k\rho}{(s)}\,.\,\rho dv + \rho \sin v\,.\cos(p-\mathrm{P})\,.\,57{\cdot}3\,dk \\ &\quad + \rho \sin v\,.\sin(p-\mathrm{P})\,.\,57{\cdot}3\,d(\tan\nu) + (s)\,d\mathrm{P}\end{aligned}$$

$$\begin{aligned}\frac{57{\cdot}3}{(a)}ds &= \left(k^2 - \frac{\sin(v+\nu)\cos(v+\nu)}{\sin v\,.\cos v\,.\cos^2\nu}\right).\frac{\rho\sin v\cos v}{(s)}\,.\,\rho dv + \frac{(s)}{\rho}\,.\,57{\cdot}3\,.\,d\rho \\ &\quad + \rho\sin v\,.\sin(p-\mathrm{P})\,.\,57{\cdot}3\,dk - \rho\sin v\,.\cos(p-\mathrm{P})\,.\,57{\cdot}3\,d(\tan\nu) \\ &\quad + (s)\,\frac{57{\cdot}3}{(a)}\,.\,d(a).\end{aligned}$$

The equations of condition between variations of the computed coordinates and the corresponding variations of the elements of the orbit become accordingly:

$$\begin{aligned}(s)\,\delta p &= \frac{k}{(s)}\,.\cos\phi\,[\delta\mu_0 + (t-t_0)\,\delta n] \\ &\quad + \frac{\rho}{(s)}\,.\,(\sin v\cos\phi + \sin\epsilon)\,.\,\delta\phi \\ &\quad + \rho\sin v\,.\cos(p-\mathrm{P})\,.\,\delta k \\ &\quad + \rho\sin v\,.\sin(p-\mathrm{P})\,.\,\delta(\tan\nu) \\ &\quad + (s)\,.\,\delta\mathrm{P}\end{aligned}$$

$$\begin{aligned}\frac{57{\cdot}3}{(a)}\,\delta s &= \Big[\left(k^2 - \frac{\sin(v+\nu)\cos(v+\nu)}{\sin v\,.\cos v\,.\cos^2\nu}\right)\frac{\cos v\cos\phi}{(s)} + \tan\phi\,.\,\frac{(s)}{\rho}\Big]\sin v \\ &\qquad\qquad [\delta\mu_0 + (t-t_0)\,\delta n] \\ &\quad + \Big[\left(k^2 - \frac{\sin(v+\nu)\cos(v+\nu)}{\sin v\,.\cos v\,.\cos^2\nu}\right)(\sin v\cos\phi + \sin\epsilon)\,.\,\frac{\rho\sin v}{(s)} \\ &\qquad\qquad - \cos\phi\,.\,\frac{(s)}{\rho}\Big]\cos v\,.\,\delta\phi \\ &\quad + \rho\sin v\,.\sin(p-\mathrm{P})\,.\,\delta k \\ &\quad - \rho\sin v\,.\cos(p-\mathrm{P})\,.\,\delta(\tan\nu) \\ &\quad + (s)\,\frac{57{\cdot}3}{(a)}\,.\,\delta(a)\end{aligned}$$

In case the eccentricity of the orbit is not large enough to allow the position-angle of the periastre to be safely treated as one of the elements, the most suitable new elements will be found by reckoning the orbital longitudes from the point of the orbit, for which $p = 0°$, or where the orbit crosses the plane of the declination circle to the north of the chief star. If ω' is the orbital longitude of the periastre reckoned from this point, or $-\omega'$ the true anomaly of this point, the values of ω' and of the correspond-

1887MNRAS..47..480M

ing $\sin\sigma$, say $\sin\sigma'$, are derived from the equations, which give p and $\sin\sigma$,

$$\sin\sigma\sin(p-\Omega)=\sin(v+\omega)\cos i$$
$$\sin\sigma\cos(p-\Omega)=\cos(v+\omega)$$

by putting

$$p=0^\circ \text{ and for } v \text{ its special value} -\omega'.$$

Thus are found

$$\sin\sigma'\sin\Omega=\sin(\omega'-\omega)\cos i$$
$$\sin\sigma'\cos\Omega=\cos(\omega'-\omega)$$
$$\sin^2\sigma'=1-\sin^2 i\sin(\omega'-\omega)\cos(\omega'-\omega).$$

The substitution of these values in the preceding or their equivalent equations

$$\sin\sigma\sin p=\sin(v+\omega)\cos i\cos\Omega+\cos(v+\omega)\sin\Omega$$
$$\sin\sigma\cos p=\cos(v+\omega)\cos\Omega-\sin(v+\omega)\cos i\sin\Omega$$

leads to the equations

$$\frac{\sin\sigma}{\sin\sigma'}\,.\sin p=\sin(v+\omega')\,.\,\frac{\cos i}{1-\sin^2 i\,.\,\sin^2(\omega'-\omega)}$$
$$\frac{\sin\sigma}{\sin\sigma'}\,.\cos p=\cos(v+\omega')+\sin(v+\omega')\,.\,\frac{\sin^2 i\,.\,\sin(\omega'-\omega)\cos(\omega'-\omega)}{1-\sin^2 i\,.\,\sin^2(\omega'-\omega)}\,.$$

By the introduction of ω' and of

$$\frac{\cos i}{1-\sin^2 i\,.\,\sin^2(\omega'-\omega)}=k' \text{ and } \frac{\sin^2 i\,.\,\sin(\omega'-\omega)\cos(\omega'-\omega)}{1-\sin^2 i\,.\,\sin^2(\omega'-\omega)}=\tan\nu',$$

as the new elements, which supplant Ω, ω, i, the equations for finding p and s become, if (σ) is written instead of

$$\frac{\sin\sigma}{\sin\sigma'}$$

and a' instead of $a\sin\sigma'$ or $a\sqrt{1-\sin^2 i\,.\,\sin^2(\omega'-\omega)}$,

$$(\sigma)\sin p=\sin(v+\omega')\,.\,k'$$
$$(\sigma)\cos p=\cos(v+\omega')+\sin(v+\omega')\,.\,\tan\nu'=\cos(v+\omega'-\nu')\sec\nu'$$
$$s=a'\rho(\sigma).$$

The differentiation of these equations gives

$$\rho(\sigma)dp=\frac{k'}{(\sigma)}\,.\,\rho\,d(v+\omega')+\rho\sin(v+\omega')\cos p\,.\,57{\cdot}3\,dk'$$
$$-\rho\sin(v+\omega')\sin p\,.\,57{\cdot}3\,d(\tan\nu')$$

$$\frac{57{\cdot}3}{a'}\,.\,ds=\left(k'^2-\frac{\sin(v+\omega'-\nu')\cos(v+\omega'-\nu')}{\sin(v+\omega')\cos(v+\omega')\cos^2\nu'}\right)\,.\,\frac{\sin(v+\omega')\cos(v+\omega')}{(\sigma)}$$
$$\rho\,d(v+\omega').$$
$$+(\sigma)\,.\,57{\cdot}3\,d\rho+\rho\sin(v+\omega')\sin p\,.\,57{\cdot}3\,dk'+\rho\sin(v+\omega')\cos p\,.\,57{\cdot}3\,d(\tan\nu')$$
$$+\ \rho(\sigma)\,.\,\frac{57{\cdot}3}{a'}\,.\,da\,.$$

1887MNRAS..47..480M

But

$$\begin{aligned}\rho d\,(v+\omega') = \frac{\cos\phi}{\rho} d\,(\mu+\omega') - (\cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi)\,.\,\tan\phi d\omega' \\ + (\sin v + \sin\epsilon\sec\phi)\,.\,\cos\phi d\phi\end{aligned}$$

and

$$57{\cdot}3\, d\rho = \tan\phi\sin v\, d\,(\mu+\omega') - \sin v\,.\,\tan\phi d\omega' - \cos v\,.\,\cos\phi d\phi.$$

Hence the equations of condition will be

$$\begin{aligned}\rho\,(\sigma)\,\delta p = & + \frac{k'\cos\phi}{(\sigma)\,\rho}\,.\,[\delta\,(\mu_0+\omega') + (t-t_0)\,\delta n] \\ & - \frac{k'}{(\sigma)}(\cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi)\,.\,57{\cdot}3\,(\tan\phi+\delta e)\sin\delta\omega' \\ & + \frac{k'}{(\sigma)}(\sin v + \sin\epsilon\sec\phi)\,.\,57{\cdot}3\,[(\tan\phi+\delta e)\cos\delta\omega' - \tan\phi] \\ & + \rho\sin(v+\omega')\cos p\,.\,57{\cdot}3\,\delta k' \\ & - \rho\sin(v+\omega')\sin p\,.\,57{\cdot}3\,\delta\,(\tan\nu')\end{aligned}$$

and, if h is written instead of

$$k'^2\sin(v+\omega')\cos(v+\omega') - \sin(v+\omega'-\nu')\cos(v+\omega'-\nu')\sec^2\nu',$$

$$\begin{aligned}\frac{57{\cdot}3}{a'}\,\delta s = & + \left(\frac{h\cos\phi}{(\sigma)\,\rho} + (\sigma)\tan\phi\sin v\right).\,[\delta(\mu_0+\omega') + (t-t_0)\delta n] \\ & - \left[\frac{h}{(\sigma)}(\cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi) + (\sigma)\sin v\right] \\ & \qquad\qquad .\,57{\cdot}3\,(\tan\phi+\delta e)\sin\delta\omega' \\ & + \left[\frac{h}{(\sigma)}(\sin v + \sin\epsilon\sec\phi) - (\sigma)\cos v\right].\,57{\cdot}3\,[(\tan\phi+\delta e)\cos\delta\omega' - \tan\phi] \\ & + \rho\sin(v+\omega')\sin p\,.\,57{\cdot}3\,\delta k' \\ & + \rho\sin(v+\omega')\cos p\,.\,57{\cdot}3\,\delta\,(\tan\nu') \\ & + \rho\,(\sigma)\,.\,\frac{57{\cdot}3}{a'}\,.\,\delta a'.\end{aligned}$$

If the assumed eccentricity is 0, and the orbital longitude of the comes, reckoned from the point of the orbit, for which $p = 0°$, is called u', the equations for finding p and s become

$$\begin{aligned}(\sigma)\sin p &= \sin u'\,.\,k' \\ (\sigma)\cos p &= \cos u' + \sin u'\,.\,\tan\nu' = \cos(u'-\nu')\,.\,\sec\nu' \\ s &= a'\,.\,(\sigma)\end{aligned}$$

and the equations between the variations of the coordinates and the variations of the elements:

$$\begin{aligned}(\sigma)\,\delta p = & + \frac{k'}{(\sigma)}\,.\,[\delta u'_0 + (t-t_0)\,\delta n] \\ & - \frac{k'}{(\sigma)}\,.\,\cos u'\,.\,2\phi\sin\omega' \\ & + \frac{k'}{(\sigma)}\,.\,\sin u'\,.\,2\phi\cos\omega' \\ & + \sin u'\,.\,\cos p\,.\,57{\cdot}3\,\delta k' \\ & - \sin u'\,.\,\sin p\,.\,57{\cdot}3\,\delta\,(\tan\nu')\end{aligned}$$

1887MNRAS..47..480M

and, if h' is written instead of

$$k'^2 - \frac{\sin(u'-\nu')\cos(u'-\nu')}{\sin u' \cos u' \,.\, \cos^2 \nu'},$$

$$\begin{aligned}\frac{57{\cdot}3}{a'} \,.\, \delta s = &+ \frac{h'}{(\sigma)} \,.\, \sin u' \cos u' \,.\, [\delta u'_0 + (t-t_0)\,\delta n] \\ &- \left(\frac{h'}{(\sigma)} \,.\, \cos^2 u' + \tfrac{1}{2}(\sigma)\right) .\, \sin u' \,.\, 2\phi \sin \omega' \\ &+ \left(\frac{h'}{(\sigma)} \,.\, \sin^2 u' - \tfrac{1}{2}(\sigma)\right) .\, \cos u' \,.\, 2\phi \cos \omega' \\ &+ \sin u' \,.\, \sin p \,.\, 57{\cdot}3\, \delta k' \\ &+ \sin u' \,.\, \cos p \,.\, 57{\cdot}3\, \delta(\tan \nu') \\ &+ (\sigma) \,.\, \frac{57{\cdot}3}{a'} \,.\, \delta a'.\end{aligned}$$

The geometrical meaning and the mutual relations of the various arcs and angles employed will be easily recognised with the help of a diagram constructed to indicate their significance on a sphere which has the chief star of the binary system in the centre. Let E denote the astral position of the Earth (in right ascension $\alpha + 180°$ and declination $-\delta$); C that of the crossing point of the declination circle and of the circle representing the plane of projection; O that of the (positive) pole of the orbit, so that EO $= i$; ☊$'$ the position of the point of the orbit where it crosses the declination circle; ☊ that of the point where it crosses the plane of projection; P the position of the periastre; S that of the satellite when its true anomaly is v. Further, let $P_{,}$ be the point where the arc EP or its prolongation meets the circle of projection; and P$'$ and C$'$ the points where the arcs $OP_{,}$ and OC or their prolongations cross the orbit.

The various arcs and angles are

$$\begin{aligned} i &= \mathrm{EO} = \mathrm{O}\,\text{☊}\,\mathrm{E} \\ \text{☊} &= \mathrm{C}\,\text{☊} = \mathrm{CE}\,\text{☊} \\ \omega &= \text{☊}\,\mathrm{P} = \text{☊}\,\mathrm{OP} \\ v &= \mathrm{PS} = \mathrm{POS} \\ \nu &= \mathrm{P}_{,}\mathrm{P} = \mathrm{P'OP} \\ \omega' &= \text{☊}'\mathrm{P} = \text{☊}'\mathrm{OP} \\ \nu' &= \text{☊}'\mathrm{C}' = \text{☊}'\mathrm{OC}' \\ \omega' - \omega &= \text{☊}'\,\text{☊} = \text{☊}'\mathrm{O}\,\text{☊} \end{aligned}$$

$$\begin{aligned} p &= \mathrm{CES} \\ \mathrm{P} &= \mathrm{CEP} \\ \sigma &= \mathrm{ES} \\ \sigma_0 &= \mathrm{EP} \\ \sigma' &= \mathrm{E}\,\text{☊}' \\ \tau &= \text{☊}\,\mathrm{SE} \\ \mathrm{OEC} &= 90° - \text{☊} \\ \mathrm{POE} &= 90° - \omega \end{aligned}$$

The arcs between any point of the orbit and O, and between any point of the circle of projection and E, are, of course, all $= 90°$.

The values of ω, i, ☊, a, corresponding to the values of k, tan ν, P, (a), or of k', tan ν', ω', a' may be found by means of the formulæ

1887MNRAS..47..480M

$$\gamma \sin 2\omega = 2 \tan \nu \qquad \gamma' \sin 2(\omega' - \omega) = 2 \tan \nu'$$

$$\gamma \cos 2\omega = 1 - k^2 - \tan^2\nu \qquad \gamma' \cos 2(\omega' - \omega) = 1 - k'^2 - \tan^2\nu'$$

$$\sin^2 i = \frac{\gamma}{1 + \gamma \sin^2 \omega} \qquad \sin^2 i = \frac{\gamma'}{1 + \gamma' \sin^2(\omega' - \omega)}$$

$$\sin \sigma_0 \sin(P - \text{☊}) = \sin \omega \cos i \qquad \sin \sigma' \sin \text{☊} = \sin(\omega' - \omega) \cos i$$

$$\sin \sigma_0 \cos(P - \text{☊}) = \cos \omega \qquad \sin \sigma' \cos \text{☊} = \cos(\omega' - \omega)$$

$$a = \frac{(a)}{\sin \sigma_0} \qquad a = \frac{a'}{\sin \sigma'}.$$

If $k \mp 1$ and $\tan \nu$ or $k' \mp 1$ and $\tan \nu'$ are of small amount, small variations of their values produce obviously large variations in the corresponding values of ω, i, ☊. The uncertainty in which semicircle ω, ω', ☊ are to be taken arises, of course, from the uncertainty which part of the true orbit is on the positive side of the plane of projection or in front of it. If the apparent motion of the comes in the direction of *in*creasing or *de*creasing position-angles is to be treated as analogous to direct or retrograde motion of the bodies of our solar system in reference to the ecliptic, the Earth is at the positive pole of the plane of projection, and the ascending node ☊ ought to be where the comes passes from the further or negative side of the plane of projection to the positive side or to that from which the observations are taken. The uncertainty whether the position-angle ☊ belongs to the ascending or descending node must be allowed for in transferring the position of the orbit from the plane of projection to a plane parallel to the terrestrial equator, or to some other fundamental plane. If J denotes the inclination of the plane of the orbit to that of the equator, N the longitude or right ascension of the ascending node, and Q the orbital longitude of the periastre reckoned from N, the values of J, N, Q are derived from those of i, ☊, ω, and of α, δ, the right ascension and declination of the star, by means of the formulæ

$$\sin J \sin(\alpha - N) = \pm \sin \text{☊} \sin i \sin \delta + \cos i \cos \delta$$

$$\sin J \cos(\alpha - N) = \pm \cos \text{☊} \cos i$$

$$\sin J \sin(Q - \omega) = \pm \cos \text{☊} \cos \delta$$

$$\sin J \cos(Q - \omega) = \mp \sin \text{☊} \cos \delta \cos i - \sin \delta \sin i$$

$$\cos J = \pm \sin \text{☊} \cos \delta \sin i - \sin \delta \cos i$$

or by equivalent formulæ, the upper or lower signs being valid according as the value of ☊ refers to the ascending or descending node of the orbit on the plane of projection.

The apparent orbit of the comes may be laid down in the ordinary way by employing the semiaxes a and β of the apparent ellipse and their position-angle θ deduced from the elements. If Π is the position-angle of the centre of the apparent ellipse at

1887MNRAS..47..480M

the place of the chief star, and S its distance, the values of Π, S, θ, α, β may be found by the formulæ

$$\mathrm{S}\sin(\Pi-\Omega)=-ae\,.\,\sin\omega\cos i$$

$$\mathrm{S}\cos(\Pi-\Omega)=-ae\,.\,\cos\omega$$

$$(\alpha^2-\beta^2)\sin 2(\theta-\Omega)=\mathrm{S}^2\,.\,\sin 2(\Pi-\Omega)=a^2e^2\,.\,\sin 2\,\omega\cos i$$

$$(\alpha^2-\beta^2)\cos 2(\theta-\Omega)=\mathrm{S}^2\,.\,\cos 2(\Pi-\Omega)+a^2\cos^2\phi\,.\,\sin^2 i$$

$$=a^2e^2\,.\,\cos 2\,\omega+a^2\sin^2 i\,(1-e^2\cos^2\omega)$$

$$\alpha^2+\beta^2=\mathrm{S}^2+a^2\cos^2\phi\,(1+\cos^2 i)=2a^2-a^2e^2-a^2\sin^2 i\,(1-e^2\cos^2\omega).$$

It will, however, be found more instructive to compute the apparent positions corresponding to a number of points of the true orbit, and to lay them down, together with the terminal points of the axes of the apparent ellipse, which may serve in drawing the latter. The eccentric anomalies ϵ in the true orbit which give the terminal points of the axes of the apparent orbit may be found from the equation

$$\tan 2\epsilon=\frac{-\sin 2\omega\,.\,\cos\phi}{\cos 2\omega-e^2\cos^2\omega+e^2\,.\,\mathrm{cosec}^2\,i}\,;$$

or, in case k and tan ν are the given elements, from

$$\tan 2\epsilon=\frac{2\tan\nu\sec\phi}{k^2+\tan^2\nu-\sec^2\phi}\,;$$

or, if k' and tan ν' are given, from

$$\tan 2\epsilon=-\cos\phi\,.\,\frac{(k'\cos\nu')^2\sin 2\omega'-\sin 2(\omega'-\nu')}{(k'\cos\nu')^2\cos 2\omega'-\cos 2(\omega'-\nu')+e^2[(k'\cos\nu')^2\cos^2\omega'-\sin^2(\omega'-\nu')}$$

The minima and maxima of the apparent distance of the two stars, and consequently the points of the greatest and least change of position-angle, may be found most conveniently by determining the true anomalies v, which satisfy the equation

$$\frac{\sin 2(v+\omega)}{\sin(v+\omega-\omega_1)}=2e\ \mathrm{cotg}^2\,i\,.\,\frac{\cos\omega}{\cos\omega_1},$$

in which $\tan\omega_1=\tan\omega\,.\sec^2 i$, and which is to be solved indirectly.

This is a biquadratic equation, and, when developed, takes probably its simplest form if $\tan\frac{1}{2}(v+\omega)$ is made the unknown quantity. When written out in full the equation becomes

$$\tan^4\tfrac{1}{2}(v+\omega)+\frac{2\cos^2 i}{e\sin\omega}\,.\,(e\cos\omega+\tan^2 i)\,.\,\tan^3\tfrac{1}{2}(v+\omega)$$

$$+\frac{2\cos^2 i}{e\sin\omega}\,.\,(e\cos\omega-\tan^2 i)\,.\tan\tfrac{1}{2}(v+\omega)-1=0,$$

and the coefficients show that of the four roots of the equation

1887MNRAS..47..480M

two are imaginary, or else that all four roots are real, according as the values of the two terms

$$e\cos\omega+\tan^2 i \text{ and } e\cos\omega-\tan^2 i$$

have the same or opposite signs.

This biquadratic equation is identical with the equation which determines the minima and maxima of the x ordinates of those instructive curves, the "ecliptical intersects" of the orbits of planets and comets, which, as I have explained in vol. xlv. of the *Monthly Notices*, will be found very serviceable in recognising the chief peculiarities and mutual relations of the orbits, and especially helpful in solving the problem of their proximities, provided, of course, that they are accompanied by the data necessary for the purpose. If there are no imaginary roots of the equation, so that there are two minima and maxima, the intersects become specially characteristic, as is the case with those of many cometary orbits, a few instances of which may be recognised on the reduced specimen plates published in vol. xlv.

If, instead of ω and i, k and $\tan\nu$ are the given elements, the equation for finding the true anomalies v, which furnish the points of min. and max. distance from the chief star in the apparent orbit, will be

$$0=(k^2+\tan^2\nu-1)\sin 2v-2\tan\nu\cos 2v+2e[(k^2+\tan^2\nu)\sin v-\tan\nu\cos v];$$

$$\text{or, putting}\quad g\sin G=-2\tan\nu,$$

$$g\cos G=k^2+\tan^2\nu-1,$$

$$h\sin H=-\tan\nu,$$

$$h\cos H=k^2+\tan^2\nu,$$

$$\frac{\sin(2v+G)}{\sin(v+H)}=-2e\,.\,\frac{h}{g},$$

which is to be solved indirectly.

If ω', k', $\tan\nu'$ are the given elements, the corresponding formulæ are

$$g'\sin G'=2\tan\nu',$$

$$g'\cos G'=k'^2+\tan^2\nu'-1,$$

$$h'\sin H'=-\sin(\omega'-\nu')\sec\nu',$$

$$h'\cos H'=k'^2\cos\omega'-\sin\nu'\,.\,\sin(\omega'-\nu').$$

$$\frac{\sin[2(v+\omega')+G']}{\sin(v+\omega'+H')}=-2e\,.\,\frac{h'}{g'}.$$

A special case still requires to be considered—the case in which one of the components of a binary system is invisible or unobserved, so that the orbit of the visible component round the centre of gravity of the system has to be determined from its observed right ascensions and declinations referred to those of other stars. The prospect of a fair determination must remain unpromising if the observations do not extend at

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1887MNRAS..47..480M

least over a full revolution, so that an approximate value of the period, or of the annual motion n, may be inferred with some confidence. If n, e, μ_0, ω', a', k', ν' are the seven elements of the orbit, the rectangular coordinates of the visible star (in right ascension α and declination δ) referred to the centre of gravity (in right ascension A and declination D) and to the circle of declination, are given by the equations

$$\begin{aligned}15\cos\delta\,(\alpha-\mathrm{A}) &= s\sin p = a'k'\,.\,\rho\sin(v+\omega')\\ \delta-\mathrm{D} &= s\cos p = a'\sec\nu'\,.\,\rho\cos(v+\omega'-\nu').\end{aligned}$$

The values of α and δ are therefore

$$\begin{aligned}\alpha &= \mathrm{A}+\frac{a'k'}{15\cos\delta}\,.\,\rho\sin(v+\omega')\\ \delta &= \mathrm{D}+a'\sec\nu'\,.\,\rho\cos(v+\omega'-\nu').\end{aligned}$$

Introducing the eccentric anomaly ϵ, instead of the true anomaly v, and putting

$$\begin{aligned}\kappa_1\sin\omega_1 &= a'k'\,.\,\sin\omega'\\ \kappa_1\cos\omega_1 &= a'k'\,.\,\cos\omega'\cos\phi\\ \kappa^s &= \frac{\kappa_1}{15\cos\delta}\end{aligned}$$

and
$$\begin{aligned}\kappa_2\sin\omega_2 &= a'\sec\nu'\,.\,\sin(\omega'-\nu')\cos\phi\\ \kappa_2\cos\omega_2 &= a'\sec\nu'\,.\,\cos(\omega'-\nu),\end{aligned}$$

the values of α and δ become

$$\begin{aligned}\alpha &= \mathrm{A}\;-e\,.\,\kappa^s\sin\omega_1+\kappa^s\,.\,\sin(\epsilon+\omega_1)\\ \delta &= \mathrm{D}^2-e\,.\,\kappa_2\cos\omega_2+\kappa_2\,.\,\cos(\epsilon+\omega_2),\end{aligned}$$

and show that of the seven elements or functions of elements of the orbit only five (n, μ_0, e; $\kappa^s\sin\omega_1$, $\kappa^s\cos\omega_1$) can be determined from observed right ascensions alone, and five (n, μ_0, e; $\kappa_2\sin\omega_1$, $\kappa_2\cos\omega_2$) from declinations alone.

The first approximate values of the unknown quantities may be got by deducing from the observations the maxima and minima of the rectangular coordinates, with the times corresponding to them, and also the times when the coordinates, in reference to some approximate values of A and D, are 0. If, in the case of observed right ascensions, τ and τ_1 are these latter times, and θ and θ_1 the times of max. and min. of $\alpha-\mathrm{A}$, approximate values of the unknown quantities are derived from

$$\begin{aligned}&2e^\circ\sin\omega_1 = 180^\circ-(\tau_1-\tau)\,n\\ &2e^\circ\cos\omega_1 = (\theta_1-\theta)\,n-180^\circ\\ &\text{max.}-\text{min.} = 2\kappa^s\\ &\text{max.}+\text{min.} = 2\,(\delta\mathrm{A}-e\,.\,\kappa^s\sin\omega_1) = 2c^s,\end{aligned}$$

and the mean anomaly at the time τ will be

$$= -\omega_1+e^\circ\sin\omega_1.$$

1887MNRAS..47..480M

If, in the case of observed declinations, θ' and θ'_1 are the times of the max. and min. of $\delta - \mathrm{D}$, and τ' and τ'_1 the times when the difference is 0, so that the four times follow in the order θ', τ', θ'_1, τ'_1, approximate values of the unknown quantities are got from

$$2e^\circ \sin \omega_2 = 180^\circ - (\theta'_1 - \theta')\, n$$

$$2e^\circ \cos \omega_2 = (\tau'_1 - \tau')\, n - 180^\circ$$

$$\text{max.}' - \text{min.}' = 2\kappa_2$$

$$\text{max.}' + \text{min.}' = 2\,(\delta\,\mathrm{D} - e\,.\,\kappa_2 \cos \omega_2) = 2c_2$$

and the mean anomaly at the time τ' will be

$$= 90^\circ - \omega_2 - e^\circ \cos \omega_2.$$

As the observed right ascensions alone leave it uncertain whether k' and consequently κ^s is positive or negative, proper circumspection must be exercised to settle the point with the help of the observations in declination; and, if required, 180° must be added to the deduced value of ω_1, so that the longitudes in the orbit may be reckoned from the point where the orbit crosses the declination circle on the north side of the centre of gravity.

The formulæ for correcting assumed approximate values of the elements or functions of elements are, if the observations in both coordinates are treated separately :

$$\delta\,(\alpha - \mathrm{A}) = +\frac{\cos\,(\epsilon + \omega_1)}{1 - e \cos \epsilon}\,.\,\frac{\kappa^s}{57\cdot3}\left[\delta\mu_0 + (t - t_0)\,\delta n\right]$$
$$+\frac{\cos\,(\epsilon + \omega_1)}{1 - e \cos \epsilon}\,.\,\sin \epsilon\,.\,\kappa^s \delta e$$
$$+\cos \epsilon\,.\,\delta\,(\kappa^s \sin \omega_1)$$
$$+\sin \epsilon\,.\,\delta\,(\kappa^s \cos \omega_1) + \delta c^s \ldots$$

$$\delta\,(\delta - \mathrm{D}) = -\frac{\sin\,(\epsilon + \omega_2)}{1 - e \cos \epsilon}\,.\,\frac{\kappa_2}{57\cdot3}\left[\delta\mu_0 + (t - t_0)\delta n\right]$$
$$-\frac{\sin\,(\epsilon + \omega_2)}{1 - e \cos \epsilon}\,.\,\kappa_2 \delta e$$
$$-\sin \epsilon\,.\,\delta\,(\kappa_2 \sin \omega_2)$$
$$+\cos \epsilon\,.\,\delta\,(\kappa_2 \cos \omega_2) + \delta c_2 \ldots$$

The two series of results have three unknown quantities, $\delta\mu_0$, δn, δe, in common, the most probable values of which must be found by properly combining the equations of the two series.

From the values of the other quantities are found

$$a'k'\,.\,\sin \omega' = 15 \cos \delta\,.\,\kappa^s \sin \omega_1$$
$$a'k'\,.\,\cos \omega' = 15 \cos \delta\,.\,\kappa^s \cos \omega_1\,.\,\sec \phi$$

and

$$a' \sec \nu'\,.\,\sin\,(\omega' - \nu') = \kappa_2 \sin \omega_2\,.\,\sec \phi$$
$$a' \sec \nu'\,.\,\cos\,(\omega' - \nu') = \kappa_2 \cos \omega_2.$$

1887MNRAS..47..480M

If the eccentricity of the orbit is not large enough to allow $\delta\mu_0$ to be made directly one of the unknown quantities, and it is preferred to treat ω' a', k', tan ν' directly as the elements, and to determine the coordinates by

$$\begin{aligned} 15\cos\delta\,.\,(\alpha-\mathrm{A}) = x &= a'k'\,.\,\rho\sin(v+\omega') \\ \delta-\mathrm{D} = y &= a'\,.\,\rho\cos(v+\omega') + a'\,.\,\rho\sin(v+\omega')\,.\,\tan\nu' \\ &= a'\sec\nu'\,.\,\rho\cos(v+\omega'-\nu'), \end{aligned}$$

the equations of condition between the differences of the observed and computed coordinates and the corresponding variations of the elements become

$$\begin{aligned} \delta x = &+ [\cos(v+\omega') + e\cos\omega']\,.\,k'\,.\,\frac{a'\sec\phi}{57{\cdot}3}[\delta(\mu_0+\omega') + (t-t_0)\,\delta n] \\ &- [\cos(v+\omega')(\cos\epsilon\,.\,\cos\phi + \tan\tfrac{1}{2}\phi) + \cos\omega']\,.\,k'\,.\,a'(\tan\phi+\delta e)\sin\delta\omega' \\ &+ [\cos(v+\omega')\sin\epsilon - \sin\omega'\cos\phi]\,.\,k'\,.\,a'[(\tan\phi+\delta\epsilon)\cos\delta\omega' - \tan\phi] \\ &+ \frac{x}{a'k'}\,.\,\delta(a'k') + 15\cos\delta\,.\,\delta\mathrm{A}\ldots \end{aligned}$$

$$\begin{aligned} \delta y = &-[\sin(v+\omega'-\nu') + e\sin(\omega'-\nu')]\sec\nu'\,.\,\frac{a'\sec\phi}{57{\cdot}3}[\delta(\mu_0+\omega') + (t-t_0)\,\delta n] \\ &+ [\sin(v+\omega'-\nu')(\cos\epsilon\,.\,\cos\phi + \tan\tfrac{1}{2}\phi) + \sin(\omega'-\nu')]\,.\,\sec\nu' \\ &\qquad\qquad .\,a'(\tan\phi+\delta e)\sin\delta\omega' \\ &- [\sin(v+\omega'-\nu')\sin\epsilon + \cos(\omega'-\nu')\cos\phi]\,.\,\sec\nu' \\ &\qquad\qquad .\,a'[(\tan\phi+\delta e)\cos\delta\omega' - \tan\phi] \\ &+ \rho\sin(v+\omega')\,.\,a'\,\delta(\tan\nu') \\ &+ \frac{y}{a'}\,.\,\delta a' + \delta\mathrm{D}\ldots \end{aligned}$$

from which the most probable values of the corrections are to be deduced. The elements ω, i,   are then found by the formulæ previously given.

Observations of Nova Cygni, of some of the Planets, and of Comet Barnard, made at Mr. Wigglesworth's Observatory with the 15·5-inch Cooke Equatorial. By J. G. Lohse.

Nova Cygni.

1885, Sept. 1.—*Nova Cygni* is of the same brightness as the small star of the 15th magnitude north preceding of it. No change in the brightness of the other stars, which were all seen, could be made out.

Oct. 3.—*Nova Cygni* was examined with a magnifying power of 260. It has become a little brighter, and is only 0·3 mag.